On Irrelevant Literals in Pseudo-Boolean Constraint Learning

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Linear Pseudo-Boolean Constraints

A linear pseudo-Boolean (PB) constraint is of the form

$$\sum_j a_j l_j \triangle k$$

where

- $\forall j, a_j \in \mathbb{Z}$
- $\forall j, l_j$ is a literal (i.e. a Boolean value)
- $\triangle \in \{<, \leqslant, =, \geqslant, >\}$
- $k \in \mathbb{Z}$ is the degree of the constraint

Example: $3a - 2\overline{b} + c - 4d \leqslant -1$

Special Cases of PB Constraints

Normalized PB constraints

$$\sum_j a_j l_j \geqslant k \text{ with } \forall j, a_j \in \mathbb{N} \text{ and } k \in \mathbb{N}$$

Example: $3a + 2\overline{b} + c \geqslant 3$

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Clauses

$$\sum_j l_j \geqslant 1$$

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Generalized Resolution [Hooker, 1988]

Most PB solvers use the following rules to learn new constraints (a.k.a. no-goods) when they encounter a conflict, so as not to do the same mistake again

$$\frac{al + \sum_{i=1}^{n} a_i l_i \geqslant d_1 \qquad b\overline{l} + \sum_{i=1}^{n} b_i l_i \geqslant d_2}{\sum_{i=1}^{n}(ba_i + ab_i) l_i \geqslant bd_1 + ad_2 - ab} \text{ (clashing addition)}$$

$$\frac{\sum_{i=1}^{n} a_i l_i \geqslant d}{\sum_{i=1}^{n} \min(a_i, d) l_i \geqslant d} \text{ (saturation)}$$

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This proof system is (theoretically) more powerful than classical resolution: its proofs may be exponentially shorter

A Conflict Analysis with Generalized Resolution

Consider the following constraints

$$\chi_1 : \bar{a} + \bar{b} + f \geqslant 2$$
$$\chi_2 : 3\bar{x} + a + b + d + e \geqslant 4$$
$$\chi_3 : 4a + 2b + 2c + x \geqslant 5$$

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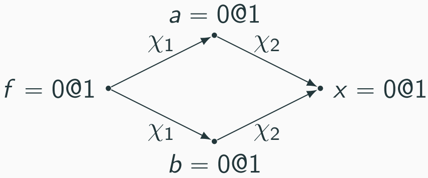

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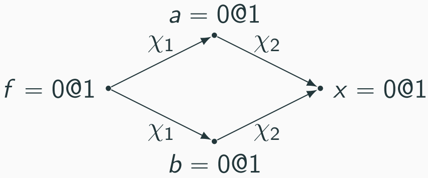

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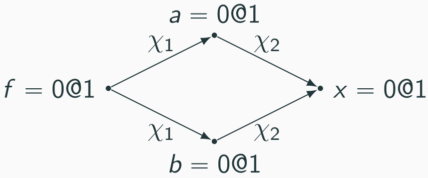

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$$\frac{\chi_3 \qquad \chi_2}{13a + 7b + 6c + d + e \geqslant 16}$$

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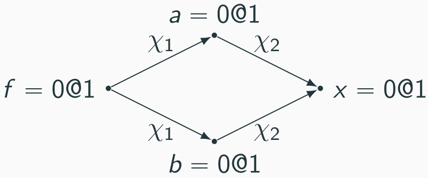

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This constraint is learned because it propagates a to 1 at level 0

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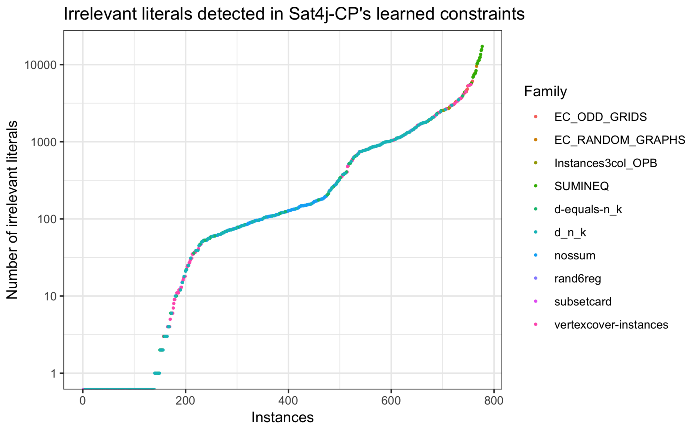
Irrelevant Literals in Practice (in Sat4j)



- Number of irrelevant literals in Sat4j-CP's first 5,000 learned constraints
- Experiments conducted on the 777 decision benchmarks from PB'16
- Sat4j as an example of Generalized-Resolution-based solver

RoundingSat's Approach [Elffers and Nordström, 2018]

RoundingSat uses a different approach, which mainly consists in using the division rule instead of saturation

$$\frac{\sum_{i=1}^{n} a_i l_i \geqslant d \qquad \alpha > 0}{\sum_{i=1}^{n} \lceil \frac{a_i}{\alpha} \rceil l_i \geqslant \lceil \frac{d}{\alpha} \rceil} \text{(division)}$$

A Conflict Analysis in RoundingSat

Consider the following constraints:

$$\chi_1 : 2\bar{c} + 2\bar{d} + b + \bar{e} \geqslant 4$$
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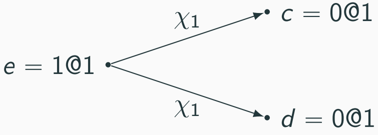

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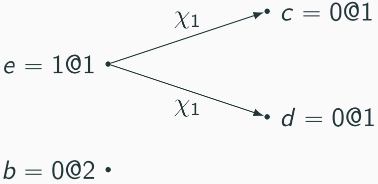

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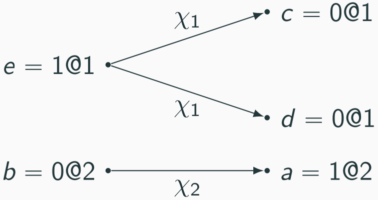

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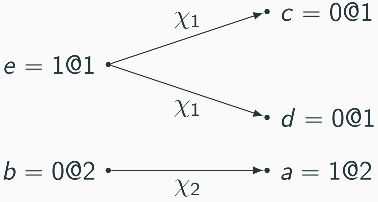

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$$\begin{tikzcd}
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$$\begin{array}{c}
\chi_2 \\
\hline
3a + 3b + c + d \geqslant 3 \\
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a + b + c + d \geqslant 1
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$$\dfrac{\dfrac{\chi_2}{3a + 3b + c + d \geqslant 3}}{a + b + c + d \geqslant 1}$$

Observe how c and d become irrelevant, and then relevant again, and how they prevent the inference of the stronger constraint $a + b \geqslant 1$

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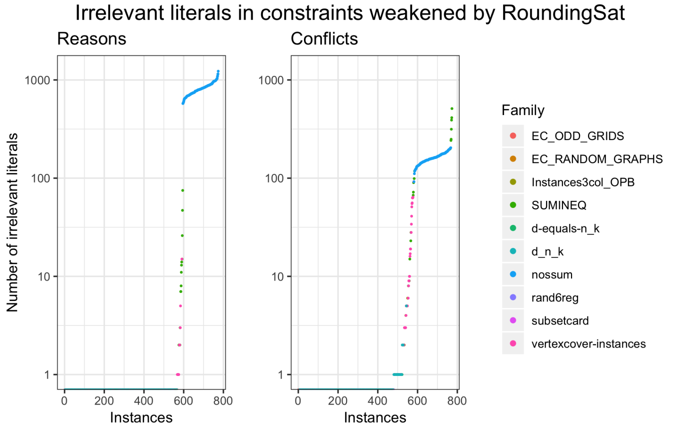

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Efficient data structures implemented in PB solvers cannot be used when cardinality constraints are hidden

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But we can still consider an incomplete approach, treating only small learned constraints, and optimizing the detection phase

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This test can be achieved by verifying that this formula is unsatisfiable

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The relevance of a literal can be checked using a PB solver

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$$13a + 7b + 6c + d = (16 - 1) = 15$$

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Another alternative to implement the relevance check is to use the dynamic programming algorithm for subset-sum

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Observe the following equation (which encodes a subset-sum problem)

$$13a + 7b + 6c + d = (16 - 1) = 15$$

If it has a solution, the corresponding model can be extended to a model of χ by satisfying e, which would hence be relevant

Note that multiple such equations may need to be considered for one literal (see next slide)

Another alternative to implement the relevance check is to use the dynamic programming algorithm for subset-sum

Its ability to compute "efficiently" all possible sums is crucial here

Detecting Irrelevant Literals: Minimizing the Checks

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If a literal is relevant, so it is for all literals with a greater coefficient

Detecting Irrelevant Literals: Algorithm

The previous observations lead to the following algorithm

Algorithm 1: detect-and-remove-irrelevant-literals

Input: A non-valid pseudo-Boolean constraint χ

Output: The constraint χ, in which all irrelevant literals are removed

foreach *coefficient c of the constraint in ascending order* **do**
    Choose a literal l having coefficient c
    if $\mathit{depends}(l, \chi)$ **then**
        return χ
    end
    Remove all literals having coefficient c from χ
    Update the degree of χ
    Saturate χ
end

Experiments

Settings

- Quadcore bi-processors Intel XEON E5-5637 v4 (3.5 GHz)
- 128 GB of memory
- 777 decision benchmarks submitted to PB'16

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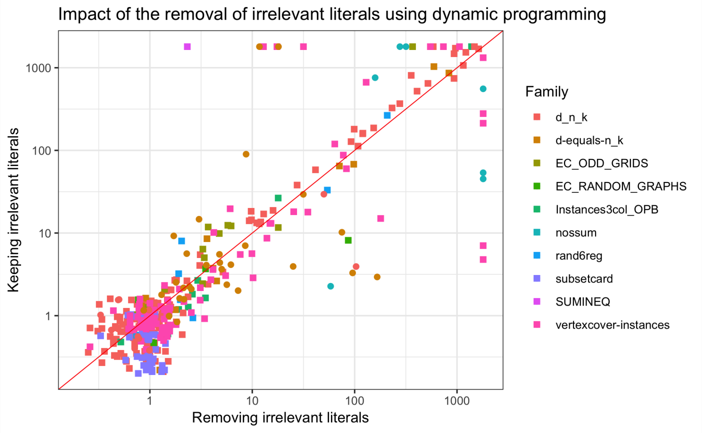
Experimented approaches

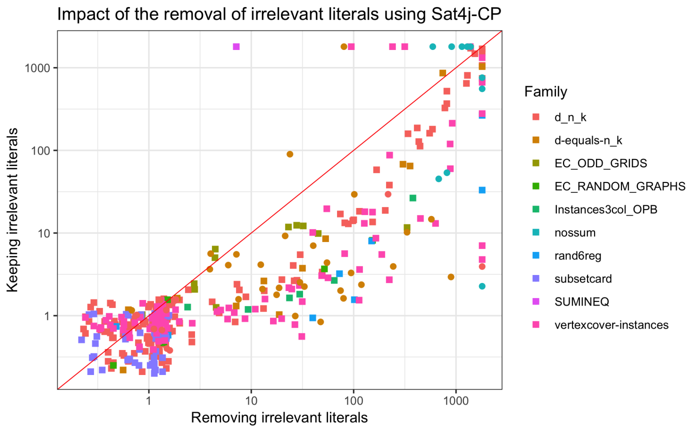
- Detection and removal implemented in Sat4j-CuttingPlanes
- Using a dynamic programming algorithm or a PB solver (5-second timeout per call)
- Only applied to learned constraints having less than 1,000 literals and a degree less than 20,000

Experimental Results: Detection using Dynamic Programming



- Instances not solved at all are not presented
- A ○ stands for `SATISFIABLE` and a □ stands for `UNSATISFIABLE`

Experimental Results: Detection with Sat4j-CP



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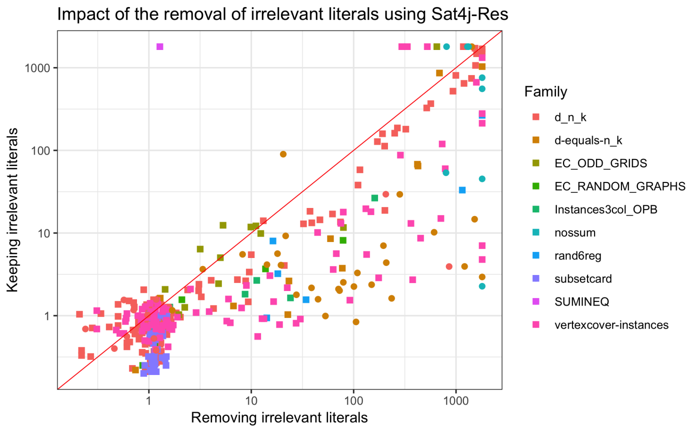
Experimental Results: Detection with Sat4j-Res



- Instances not solved at all are not presented
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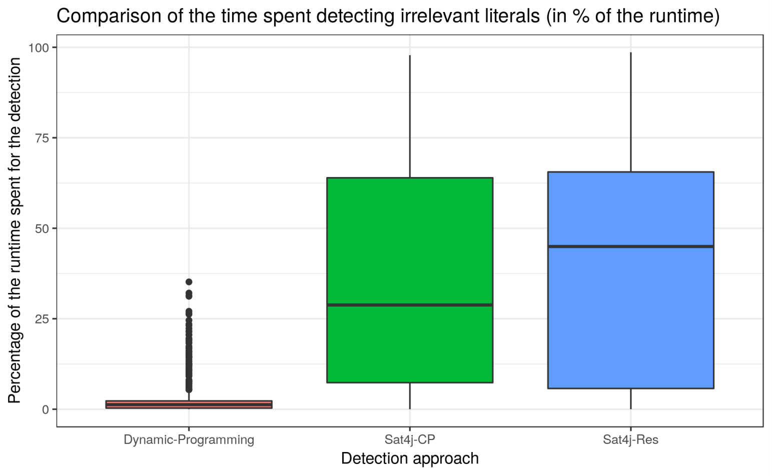
Experimental Results: Detection Runtime



Conclusion and Perspectives

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Perspectives

- Improve the detection algorithm to remove more literals
- Find a proof system guaranteeing not to infer such literals

On Irrelevant Literals in Pseudo-Boolean Constraint Learning

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