

# Flipped Recording

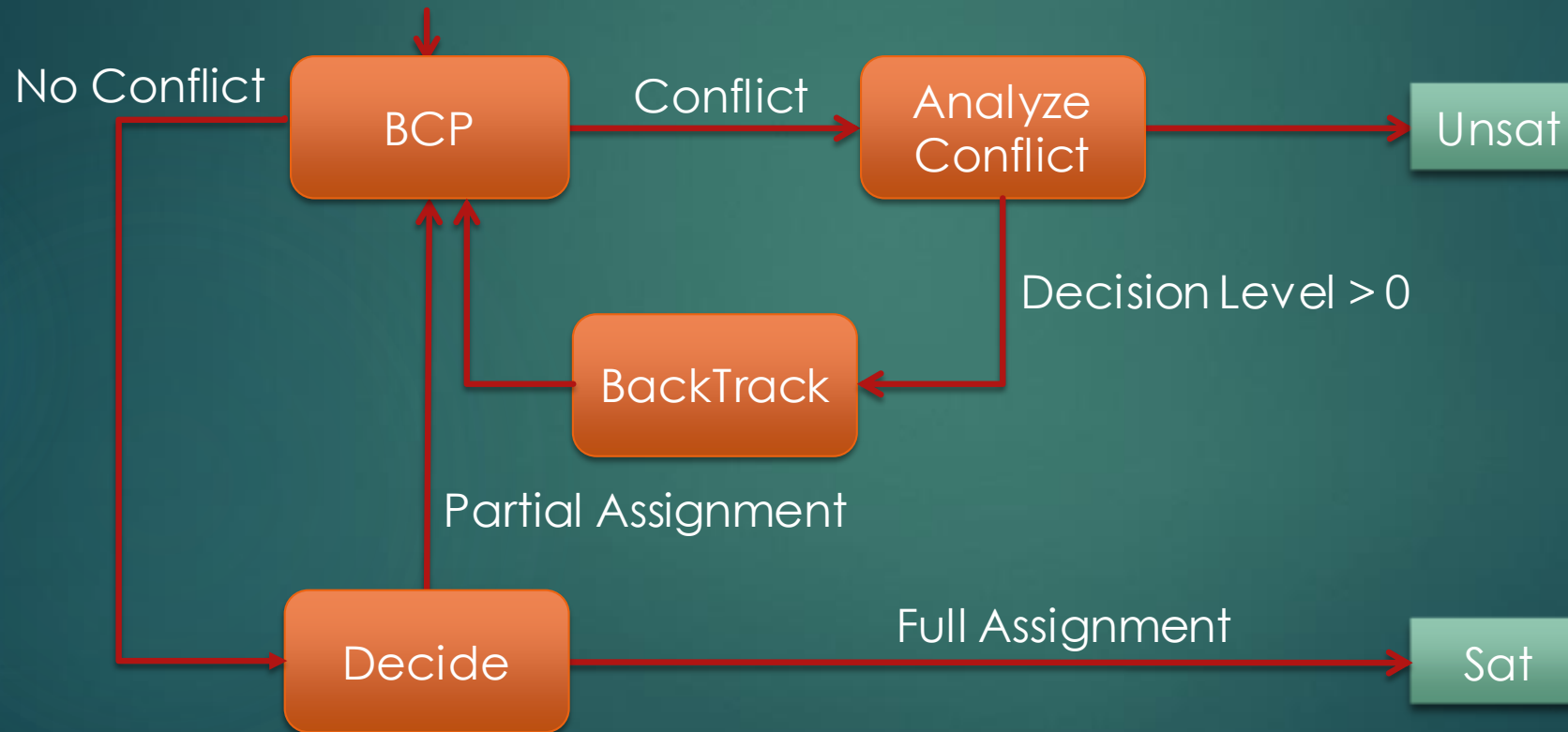
VADIM RYVCHIN (CISCO) AND ALEXANDER NADEL (INTEL)

PRAGMATICS OF SAT 2019

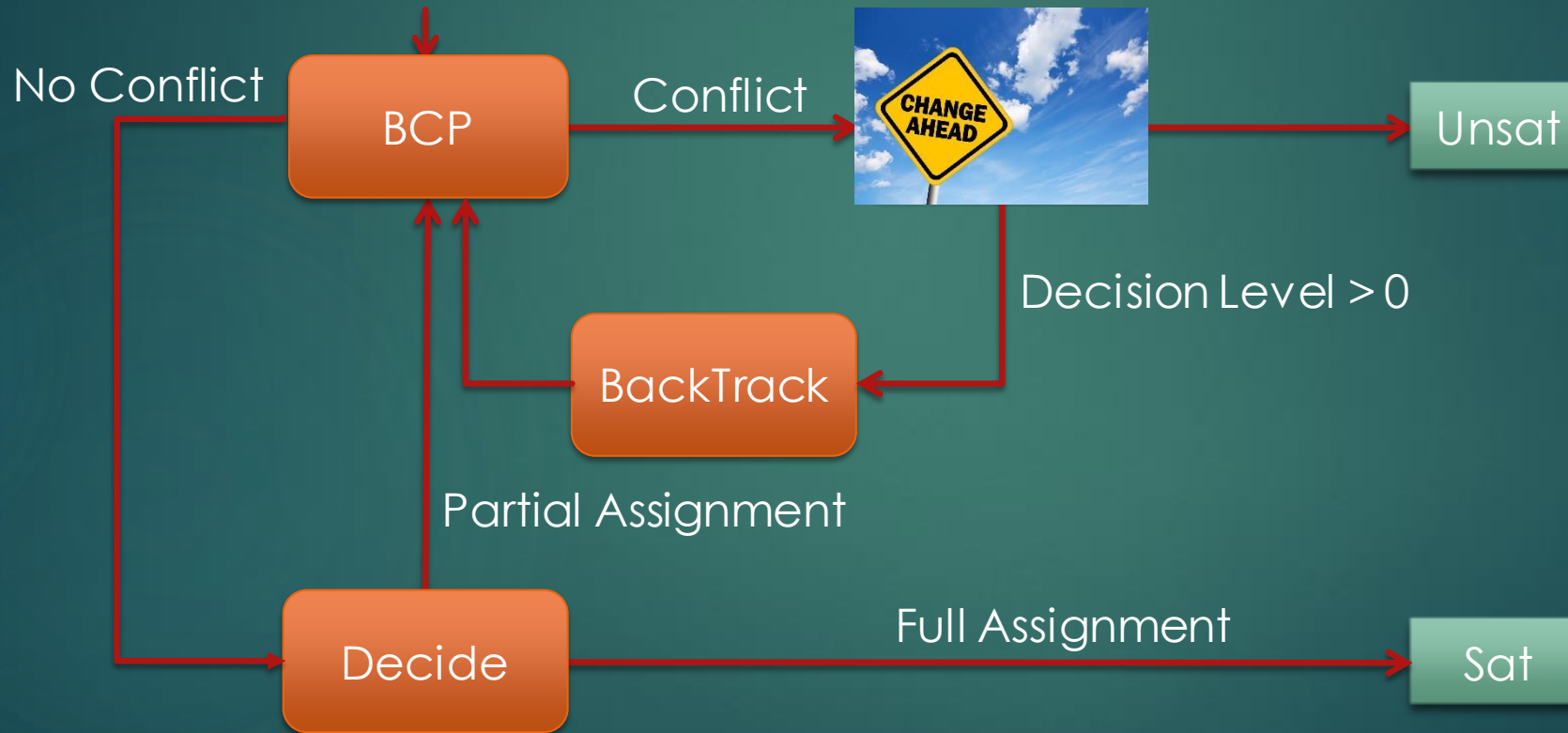


# Basic CDCL Solver

2



# Basic CDCL Solver



# Analyze Conflict

$$C_1 = (\neg v_1 + v_2)$$

$$C_2 = (\neg v_1 + v_3 + v_9)$$

$$C_4 = (\neg v_4 + v_5 + v_{10})$$

$$C_5 = (\neg v_4 + v_6 + v_{11})$$

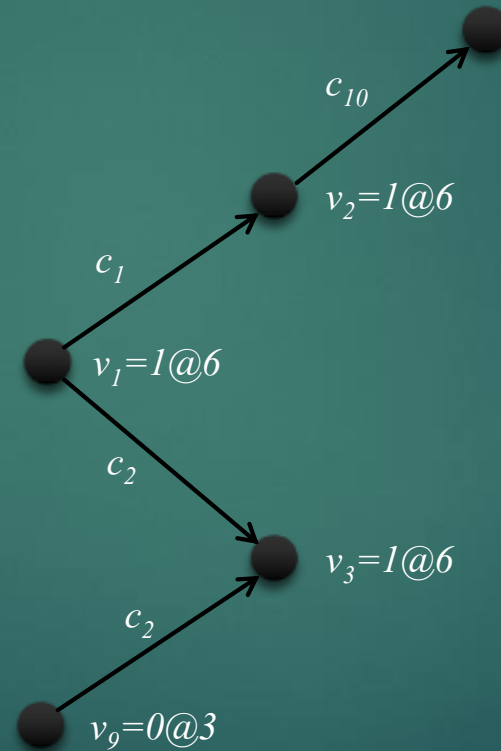
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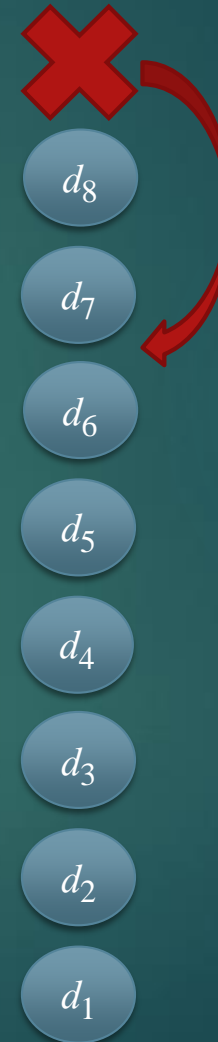
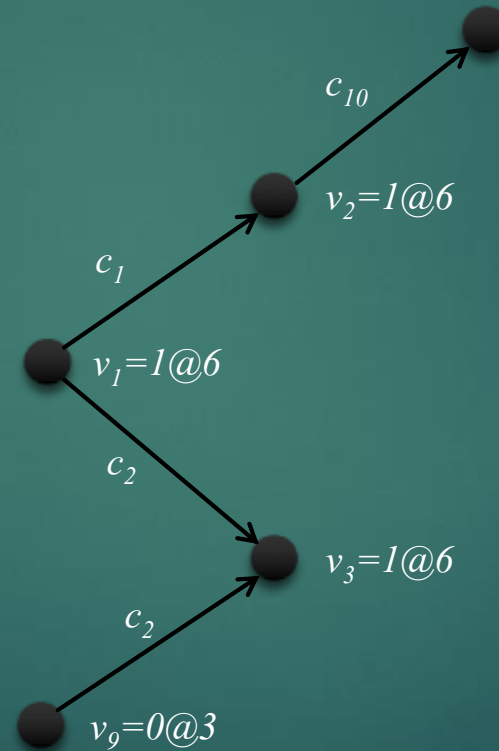
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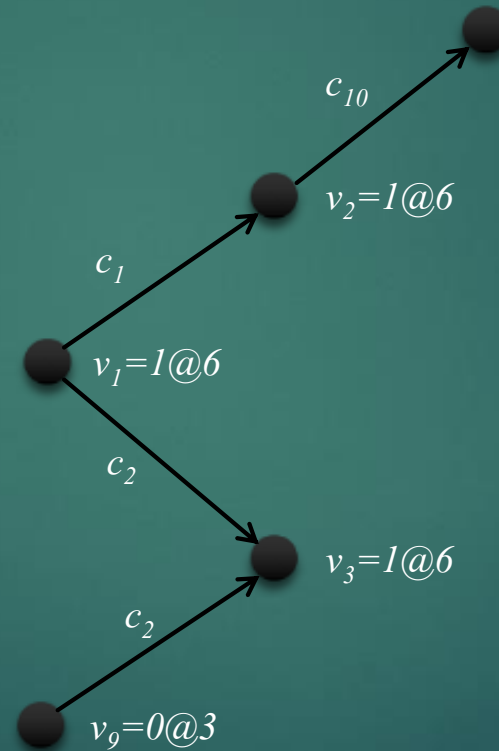
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Learnt Clause:  
 $C_3 = (\neg v_2 + \neg v_3 + v_4)$

# Analyze Conflict

## BCP

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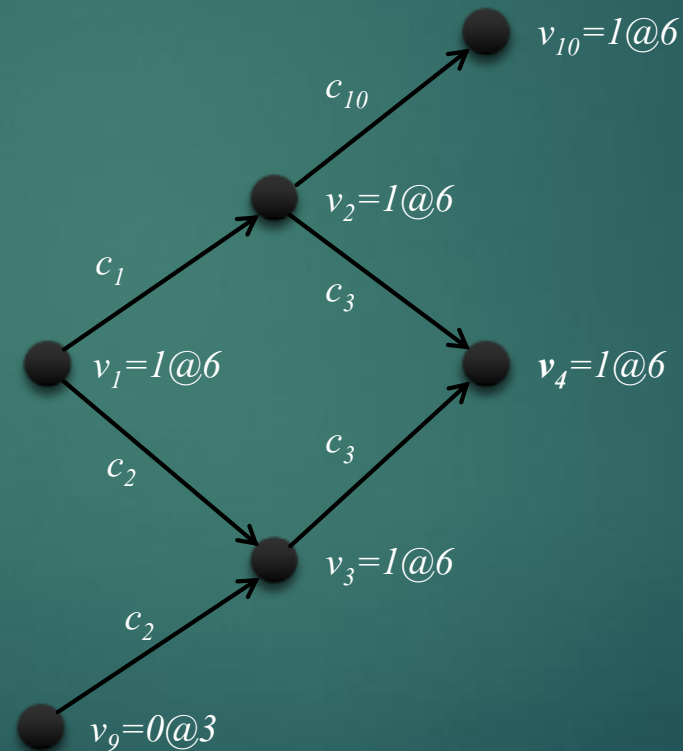
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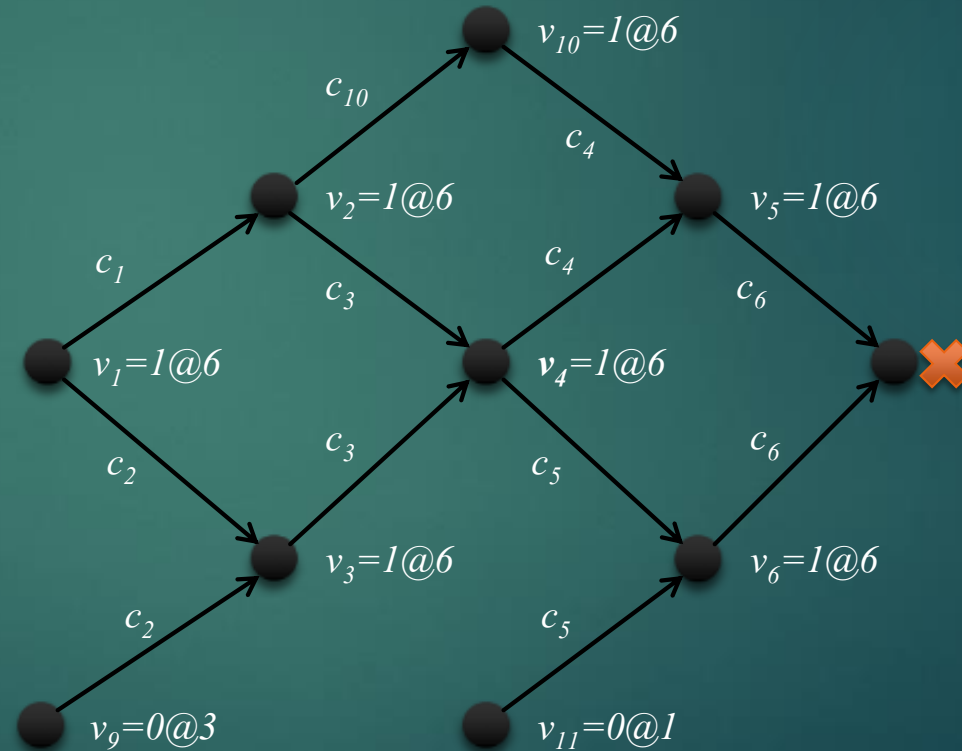
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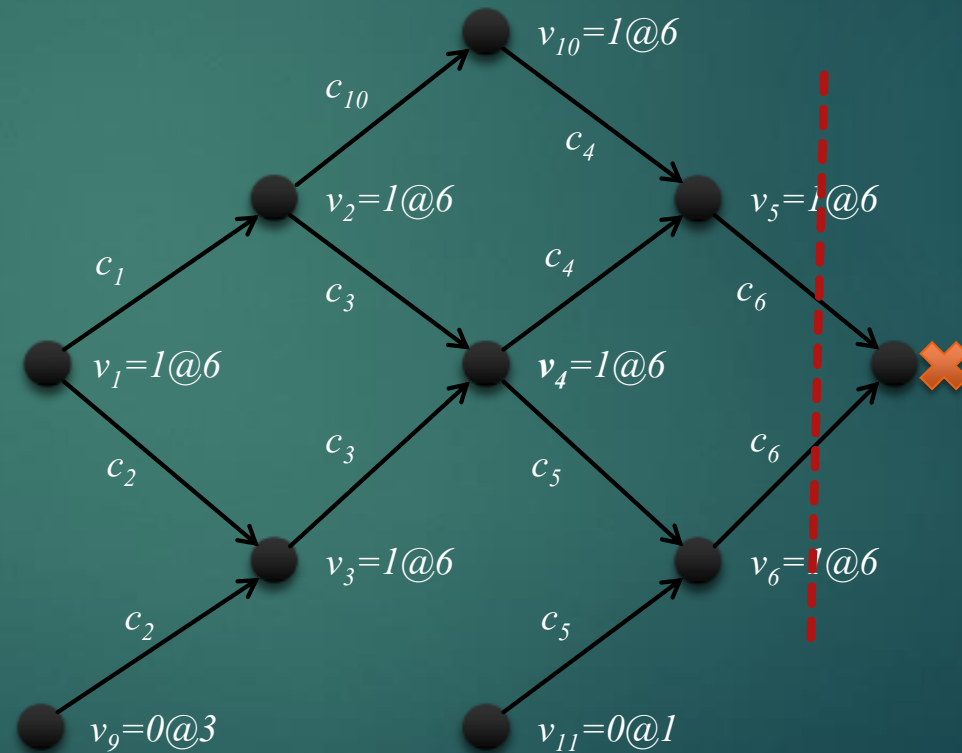
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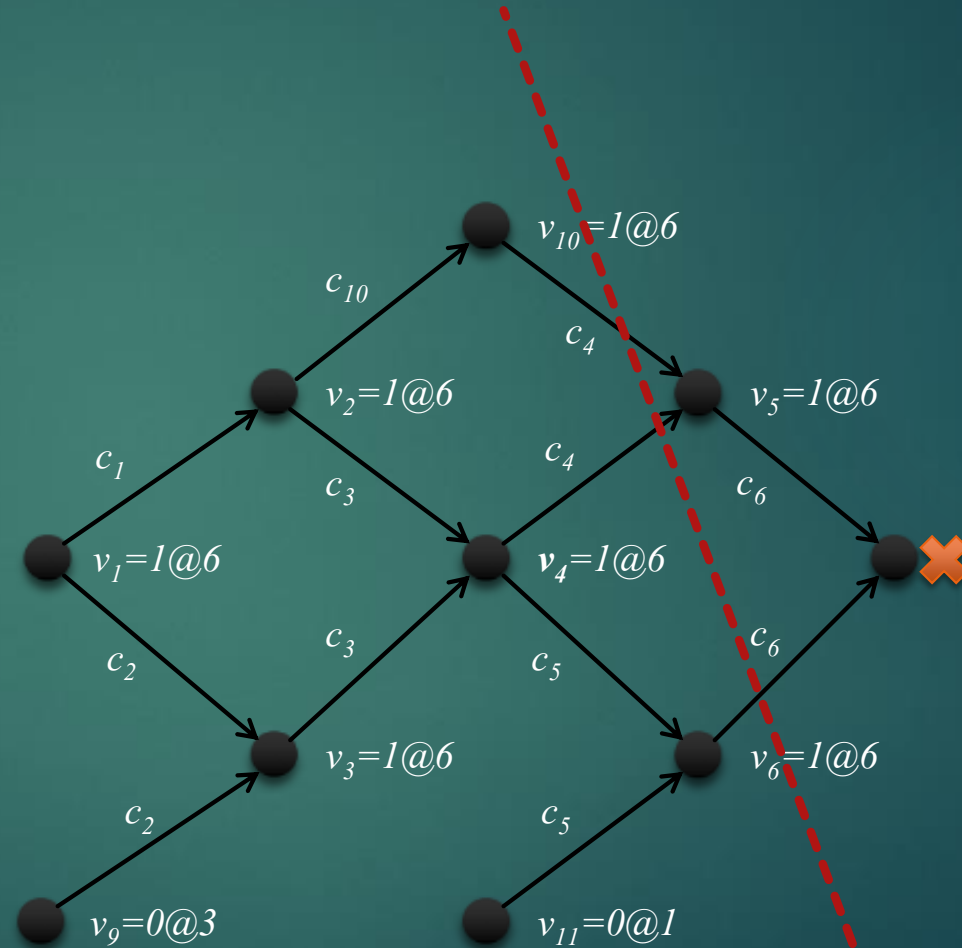
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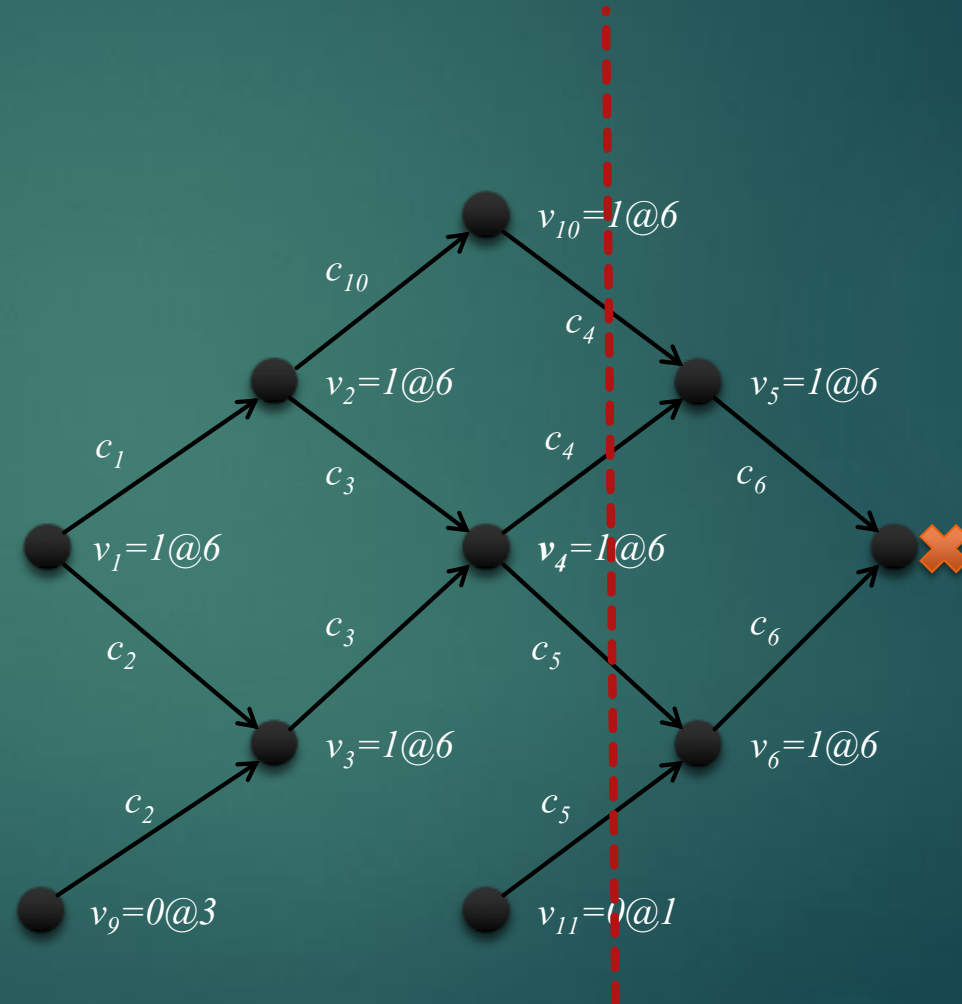
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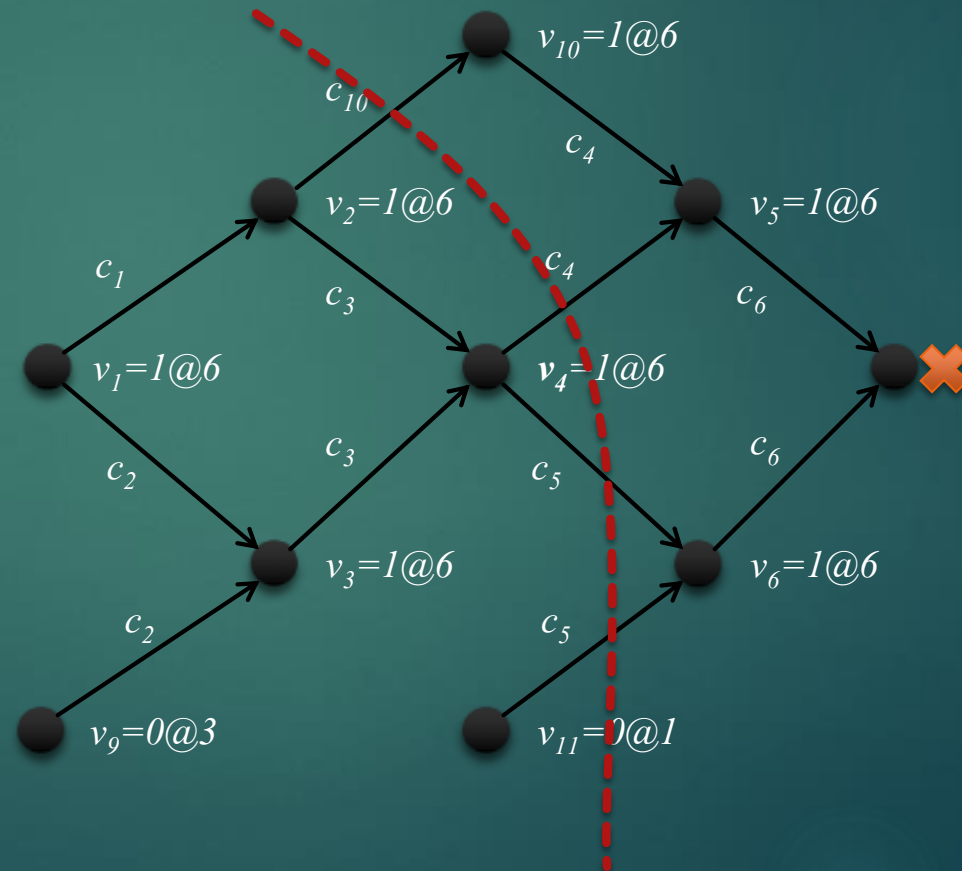
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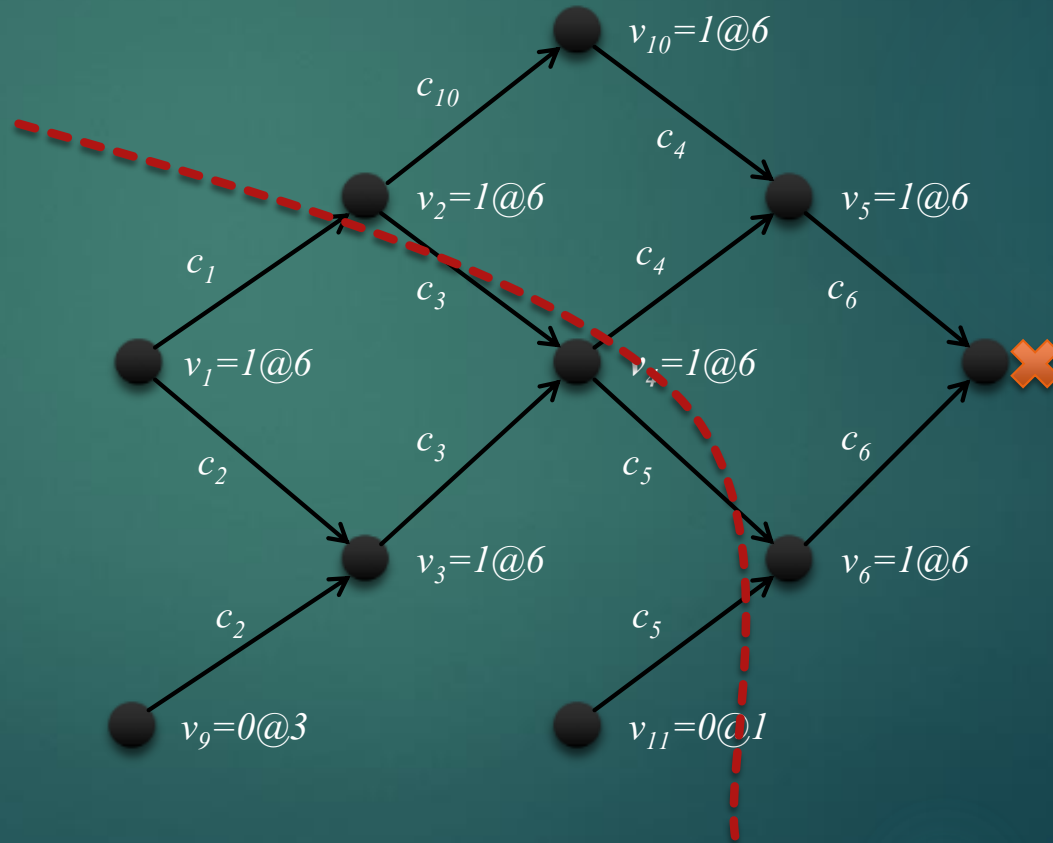
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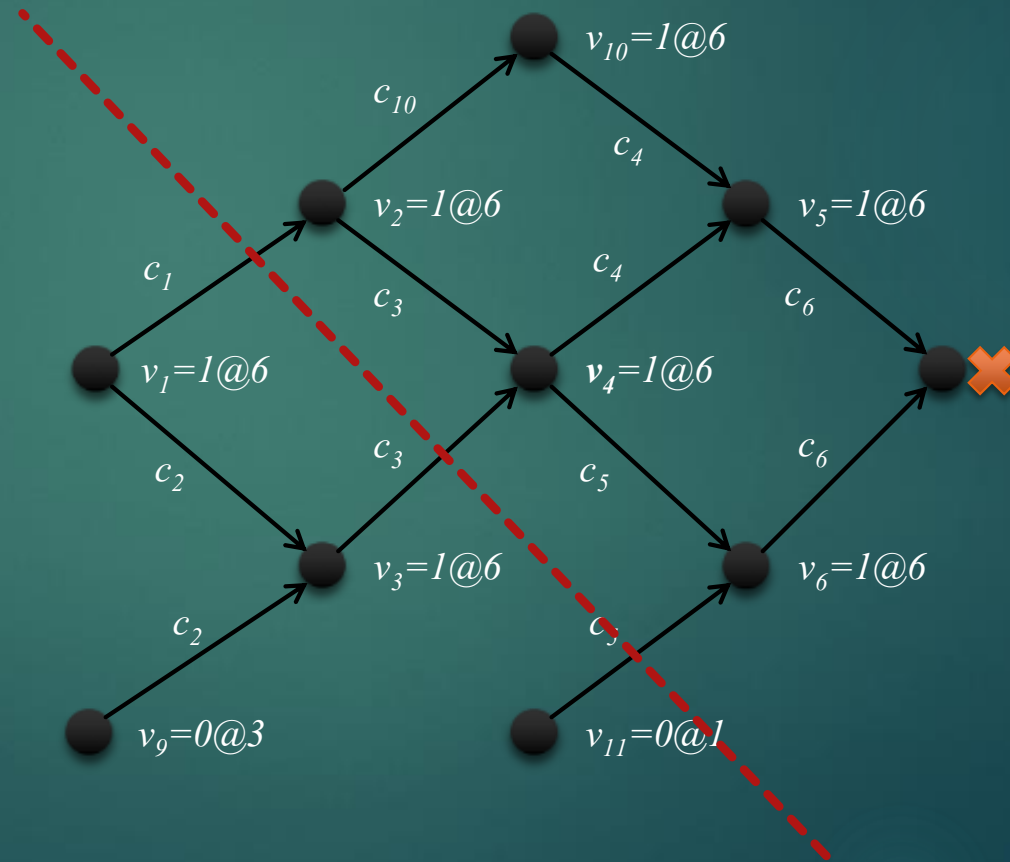
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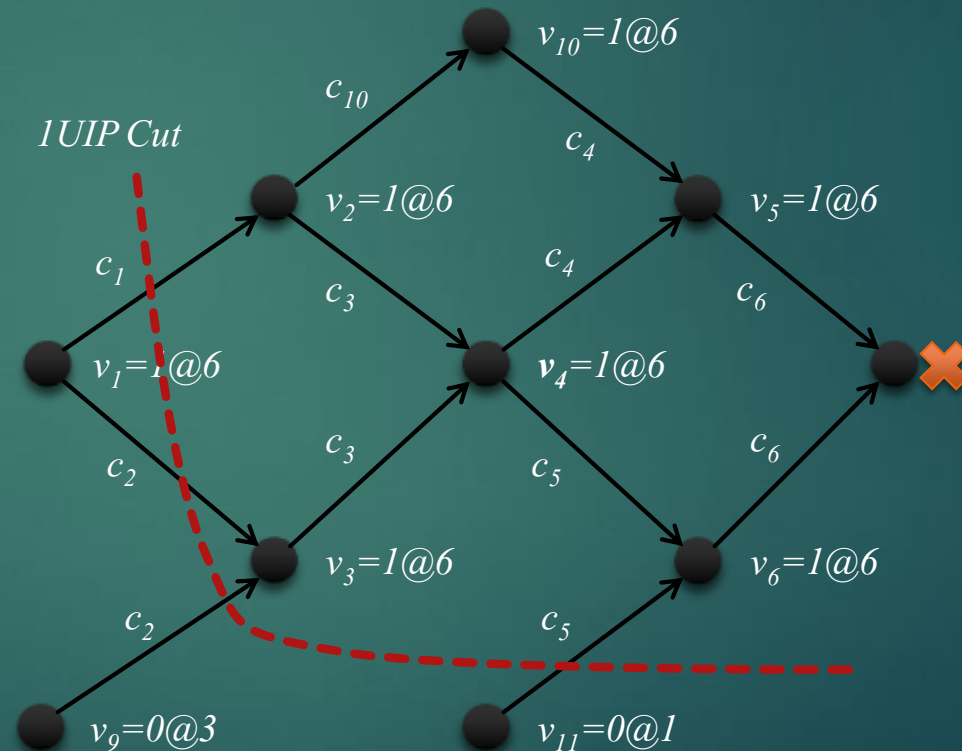
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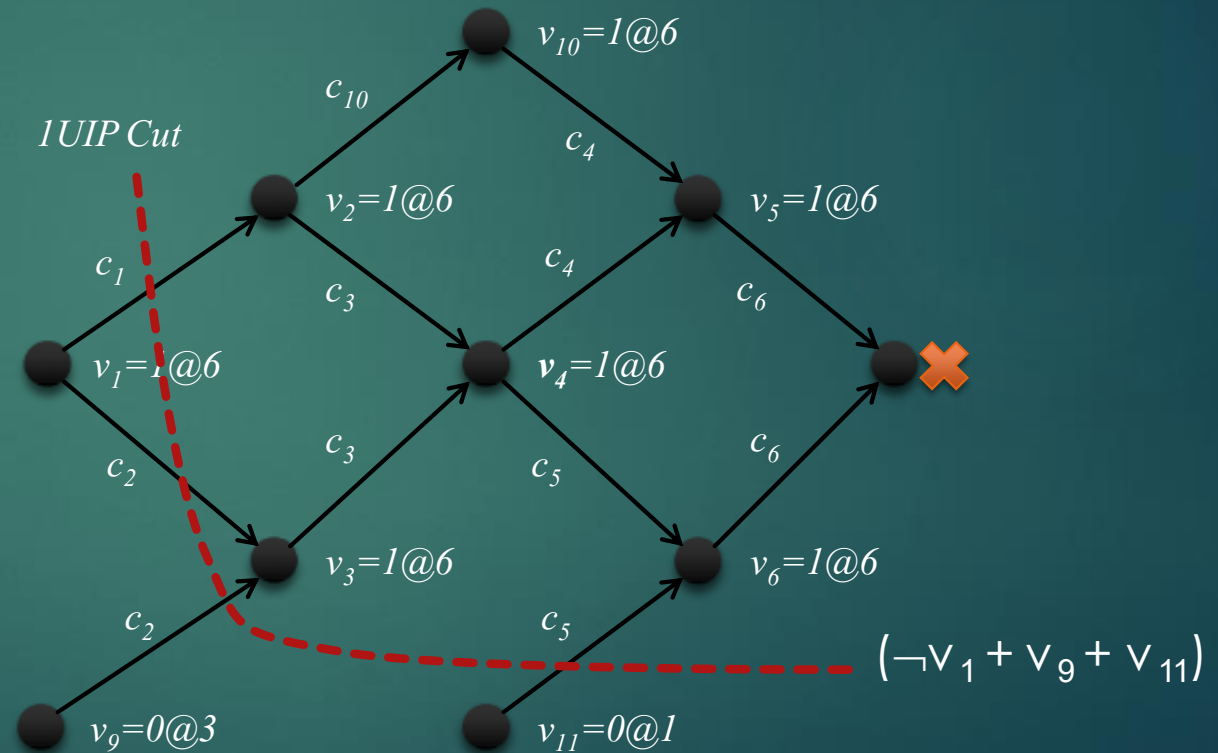
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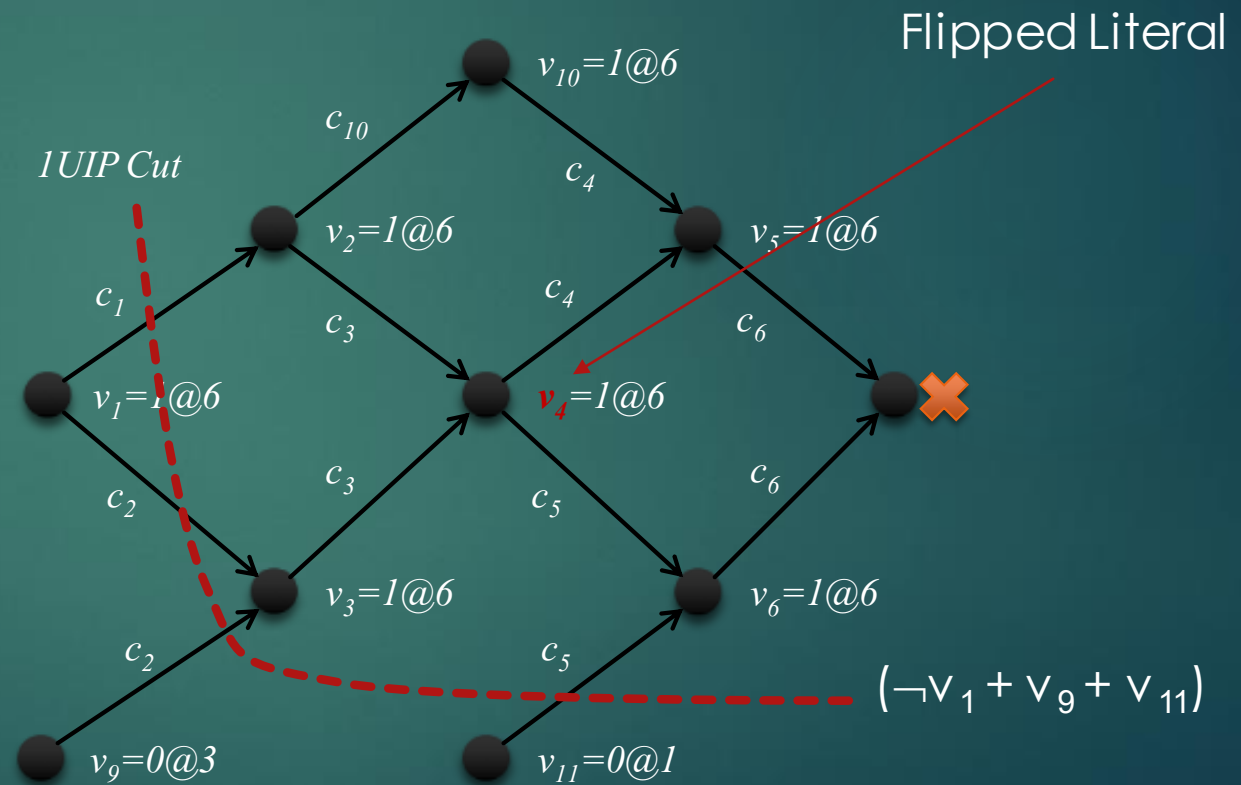
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# Analyze Conflict Flipped Literal

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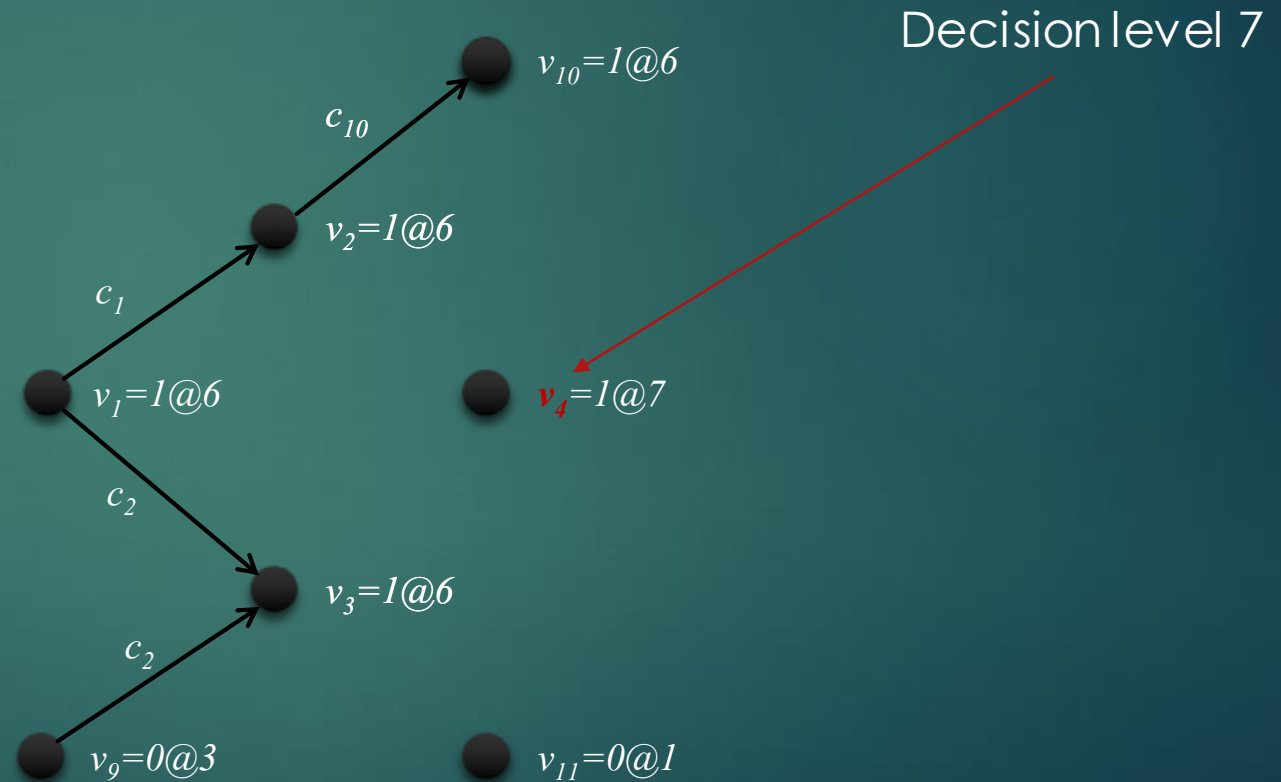
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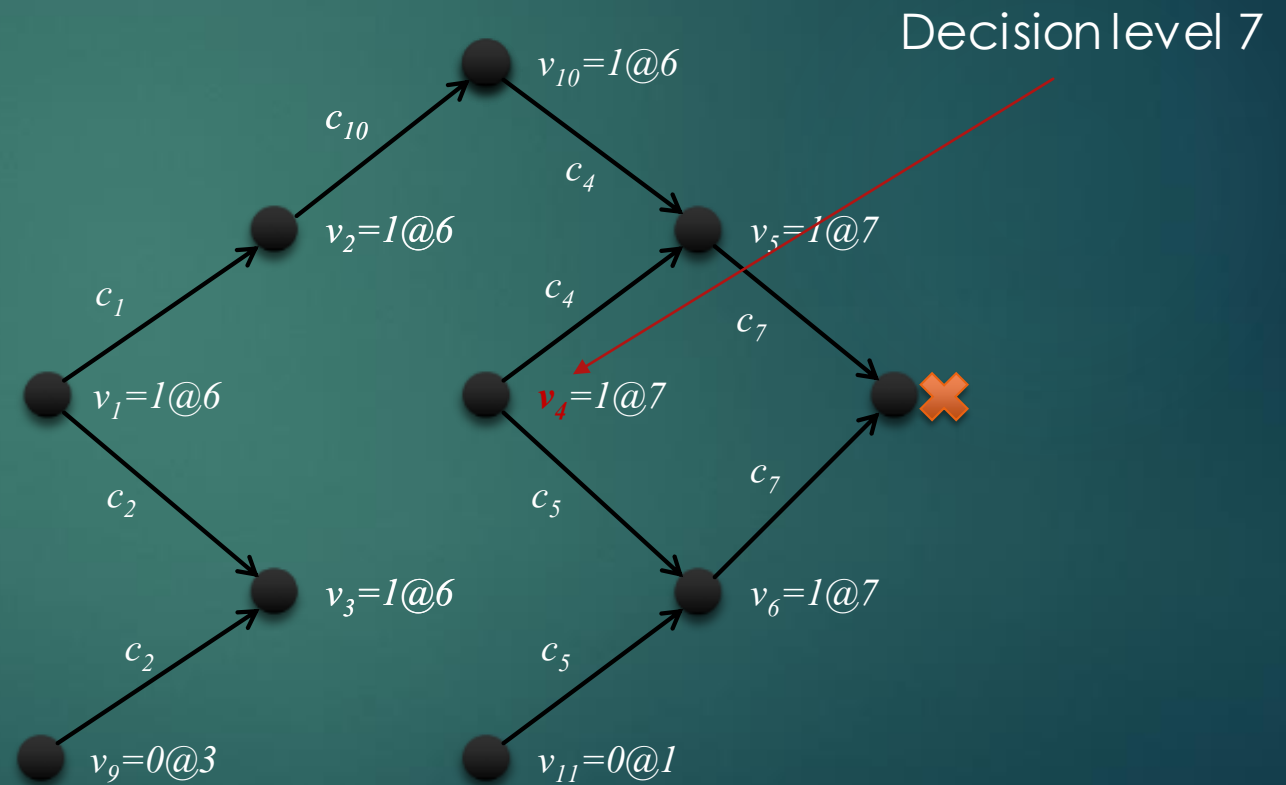
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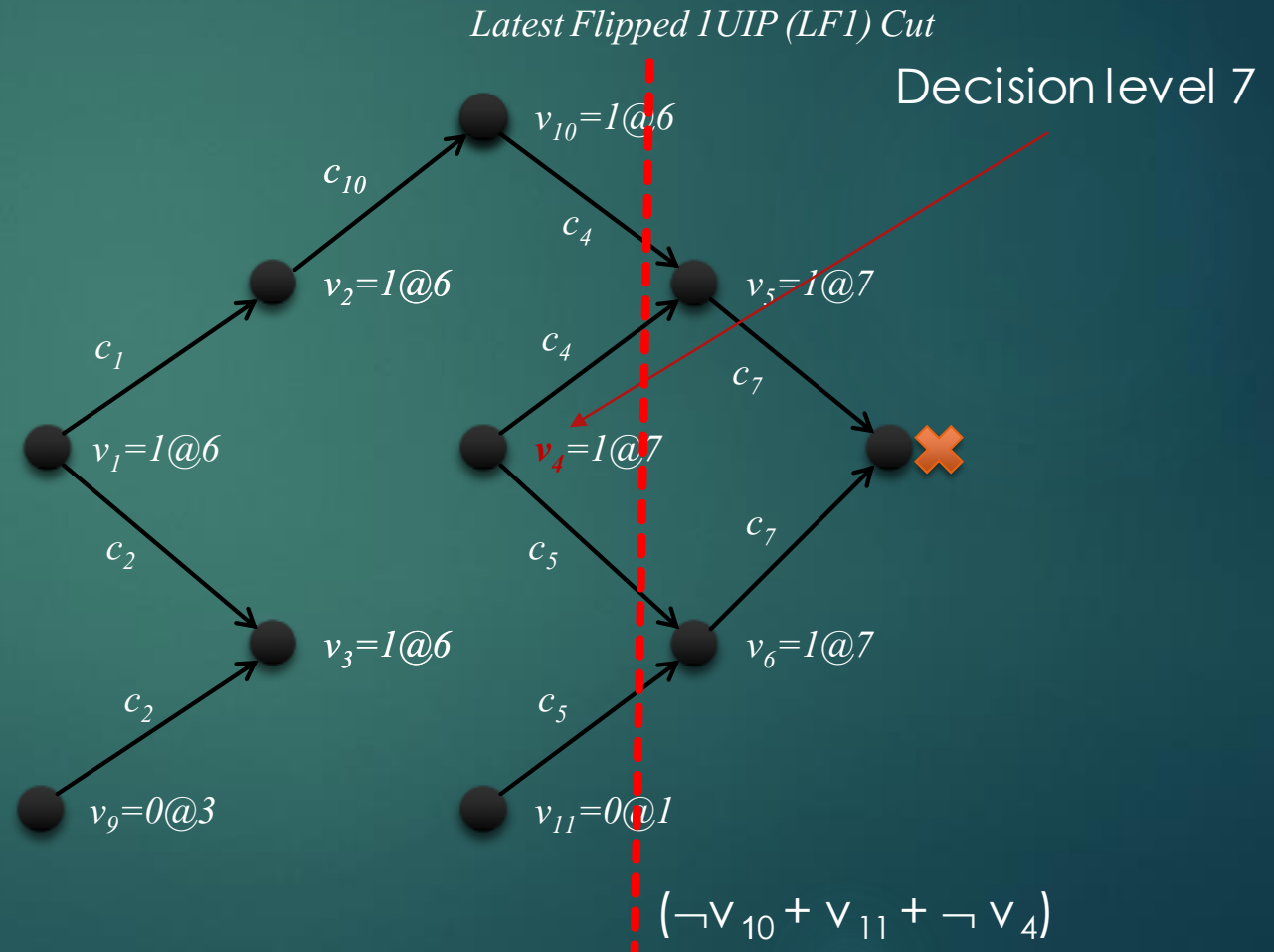
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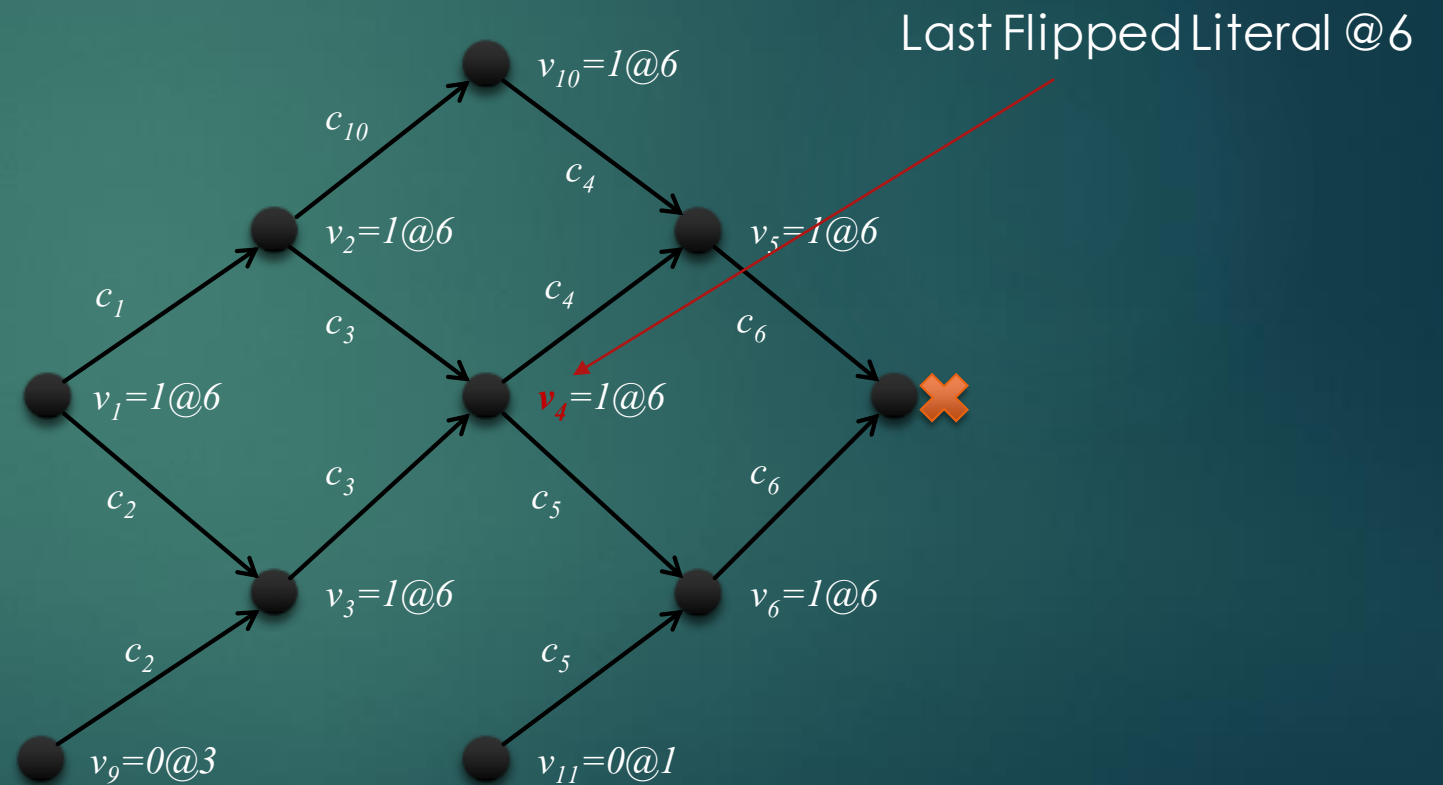
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# Analyze Conflict Combined

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$$C_3 = (\neg v_2 + \neg v_3 + v_4)$$

$$C_4 = (\neg v_4 + v_5 + v_{10})$$

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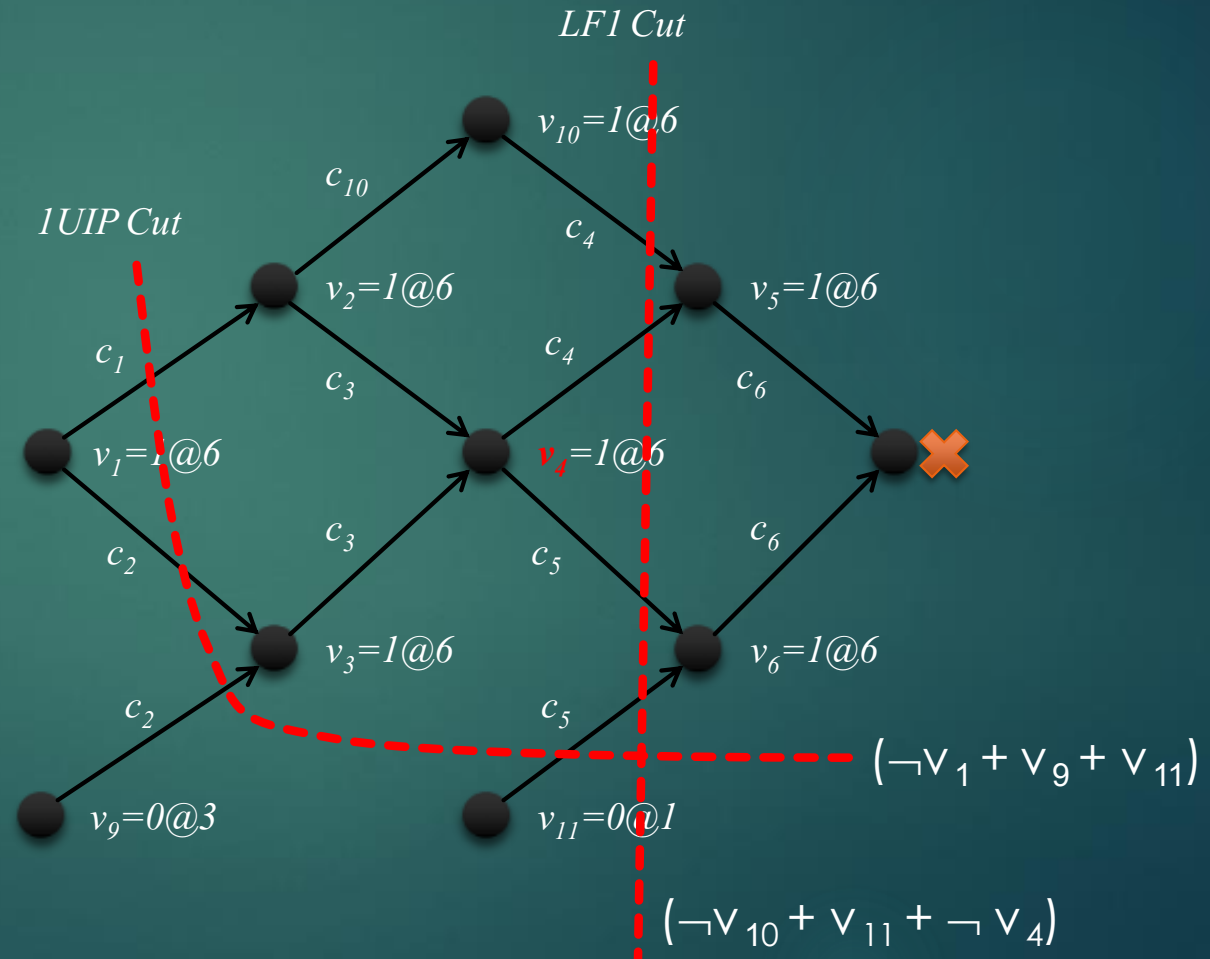
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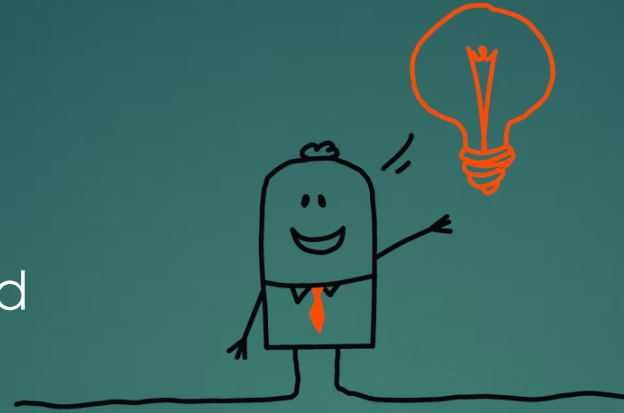
► Latest Flipped Variable: v4



# The Idea

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- ▶ Already 12 years old



- ▶ The difference in this work:
  - ▶ Much more efficient algorithm
  - ▶ Support for the CB
  - ▶ Applying multiple simplification techniques on the LF1 clause
  - ▶ Suggesting different heuristics when to learn LF1 clause

# LF1 cut

- ▶ LF1 cut performed with respect to latest flipped variable as a conflicting decision (sort of helper to a decision heuristic)
- ▶ Cut with respect to the latest flipped literal in current conflicting level
- ▶ Can be done during usual pass of a conflict analyzer
- ▶ Might be the same cut as 1UIP



# Original algorithm

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- ▶ For every decision level keep a set of all flipped literals in that level
  - ▶ Later only 1 flipped literal per level
- ▶ If there is a flipped literal then learn a new LF1 clause
  
- ▶ No simplification algorithm applied to the LF1 clause
- ▶ The clause has at least 2 literals from current decision level
- ▶ Not working in CB case as there is a mismatch between flipped literal



# New Algorithm

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- ▶ Observation 1: Only last conflict flipped literal can be activated
- ▶ Observation 2: LF1 clause is an intermediate clause in Analyze Conflict
- ▶ Keep flipped literal only for the latest conflict
- ▶ If LF1 clause  $\neq$  Learned Clause:
  - ▶ Minimize LF1 clause
  - ▶ If LF1 clause is asserting and  $LBD(\text{LF1 clause}) \leq LBD(\text{Learned Clause})$ :
    - ▶ swap(learned clause, LF1 clause)

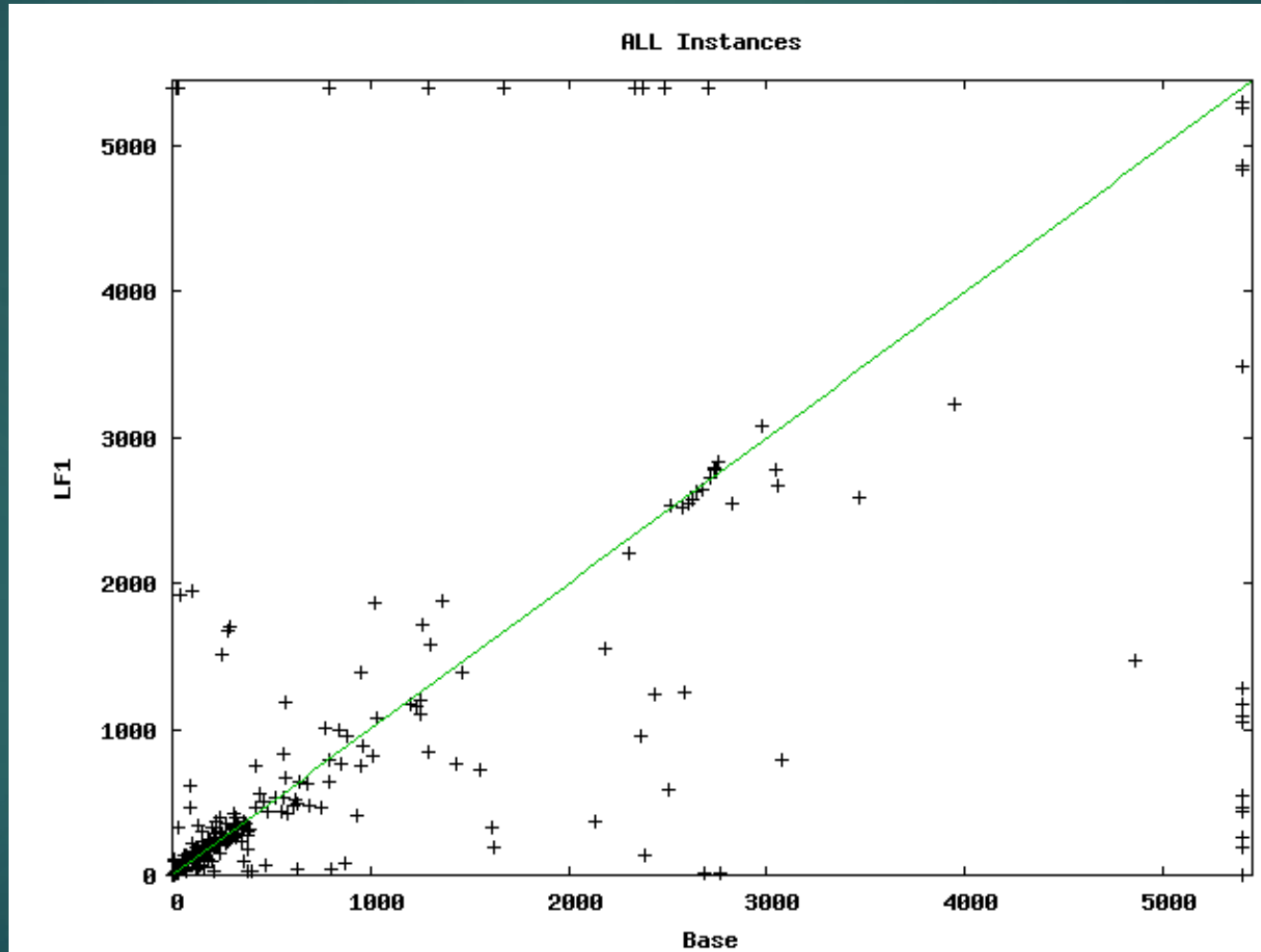


# Interesting Cases

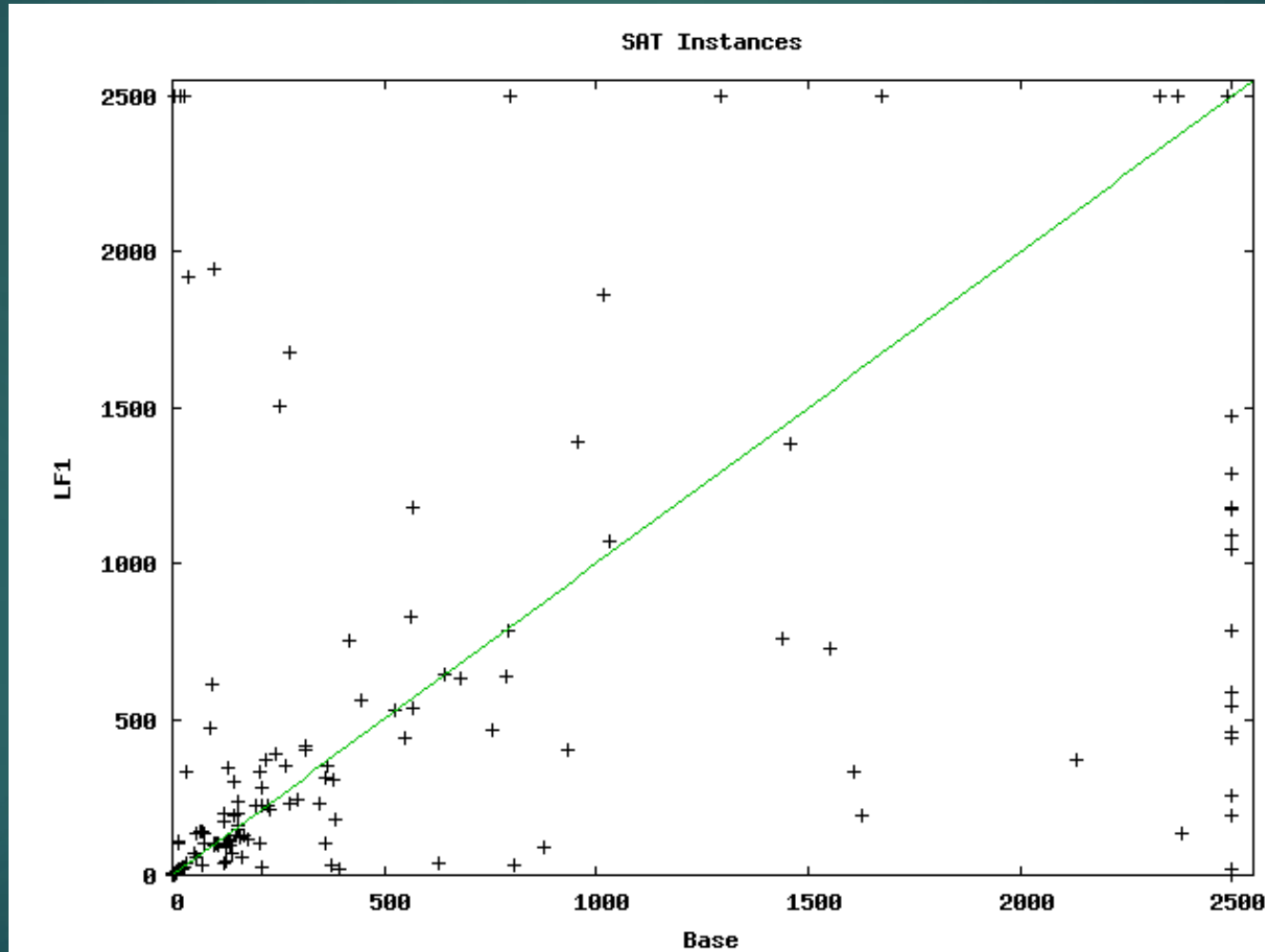
- ▶ Simplification of LF1 learned clause using binary resolution might make the clause asserting and even unit and have a better LBD score than original learned clause

# Results – SAT'18 5400 sec

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# Results – SAT'18 2500 sec



Performance						Statistics (for the timeout of 5400 sec.)			
		2500 sec.		5400 sec.		<i>LF1</i> learned		<i>LF1</i> unit clauses	
		Base	<i>LF1</i>	Base	<i>LF1</i>	Average	Median	Average	Median
SAT	Solved Time	125 88681	132 68713	142 175270	146 144166	26.8%	27.1%	40.6	10
UNSAT	Solved Time	96 45459	97 42021	103 72599	105 68079	34.5%	33.6%	255.8	69
ALL	Solved Time	221 134140	229 110734	245 247869	251 212245	30%	31.6%	130.6	22

Thanks!

