DepQBF: A Dependency-Aware QBF Solver (System Description)

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QBFEVAL’10: Overall Score-Based Ranking

Solver	*Score*
DepQBF	**2896.68**
DepQBF-pre	2508.96
aqme-10	2467.96
qmaiga	2117.55
AIGSolve	2037.22
quantor-3.1	1235.14
struqs-10	947.83
nenofex-qbfeval10	829.11

`http://www.qbflib.org/index_eval.php`

This Talk:

- DepQBF 0.1 system overview.
- Selected features: restarts, removal of learnt constraints.
- Experimental evaluation.

Overview

DepQBF:

- Input: QBFs in Prenex-CNF (PCNF).
- QDPLL with conflict-driven clause and solution-driven cube learning.
- Analysis of variable dependencies.

Variable Dependencies in QBFs:

- PCNF $Q_1 Q_2 \ldots Q_n. \phi$: linearly ordered sets of quantified variables.
- Left-to-right prefix order: strong dependencies.
- DepQBF: relaxing prefix order by dependency schemes.

Example

Quantifier ordering matters:

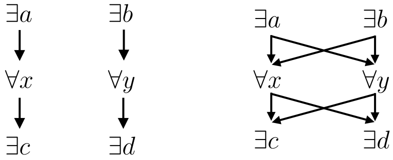
- $\forall x \exists y. (x = y)$ is satisfiable: value of y *depends* on value of x.
- $\exists y \forall x. (x = y)$ is unsatisfiable: value of y is fixed for all values of x.

Dependency Schemes

Dependency Schemes: $D \subseteq (V_\exists \times V_\forall) \cup (V_\forall \times V_\exists)$. [SS09, LB09, LB10, Ben05]

- $(x, y) \notin D$: y independent from x.
- $(x, y) \in D$: conservatively regard y as depending on x.
- DepQBF: *standard dependency scheme* $D^{\text{std}} \subseteq D^{\text{triv}}$.
  - Previous work: D^{std} as dependency-DAG over equivalence classes.
  - Efficient integration.

Example: $\exists a, b \forall x, y \exists c, d.\ (a \vee x \vee c) \wedge (a \vee b) \wedge (b \vee d) \wedge (y \vee d)$.



D^{std} D^{triv}

Standard dependency scheme D^{std}, quantifier prefix D^{triv}.

System Overview (1/7)

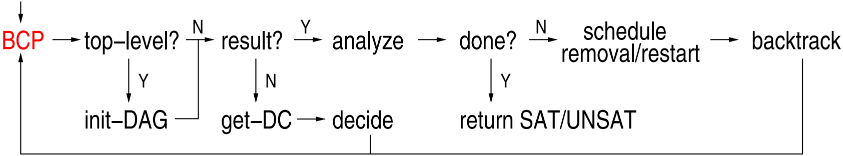


Figure: DepQBF workflow.

Boolean Constraint Propagation (BCP):

- Propagation of unit and pure literals.
- Watched data-structures for efficient detection.

System Overview (2/7)

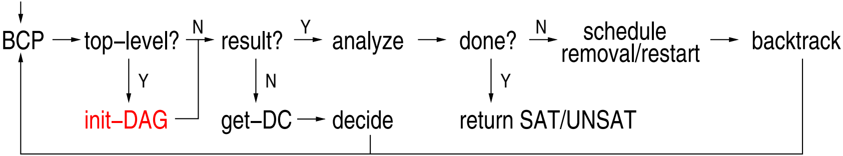


Figure: DepQBF workflow.

Initialize Dependency-DAG:

- Top-most decision level 0.
- All assignments at top-level are permanent.
- Permanent simplifications (satisfied clauses).
- Potential reduction of dependencies.

System Overview (3/7)

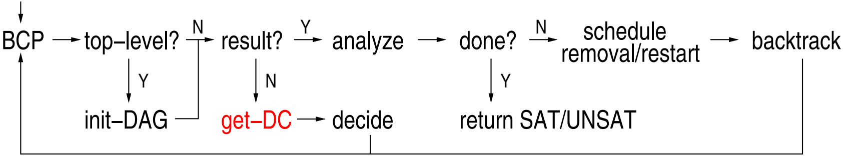


Figure: DepQBF workflow.

Retrieve Decision Candidates (DC):

- Get possible decision variables (candidates) from dependency-DAG.
- Candidate: all “preconditions” (predecessors in DAG) assigned.
- Candidate set is maintained incrementally and lazily.

System Overview (4/7)

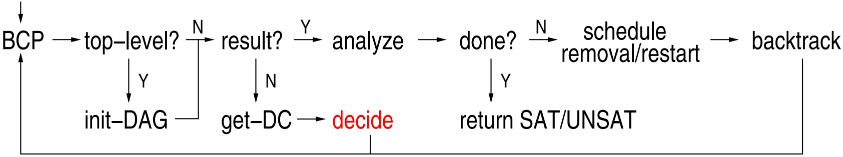


Figure: DepQBF workflow.

Decision Making:

- Select decision variable from candidate set.
- Activity-based priority queue of variables (VSIDS, like MiniSAT 2).
- Assignment caching.

System Overview (5/7)

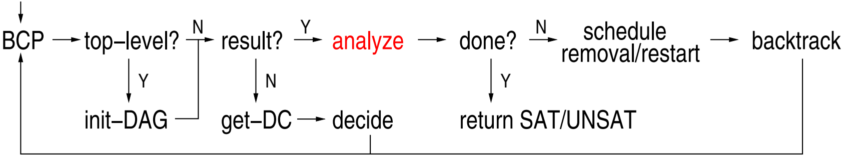


Figure: DepQBF workflow.

Constraint Learning (Result Analysis):

- Conflict/solution: generate *asserting* learnt clause/cube.
- Augmented CNF: $\phi := \phi_{OCL} \wedge (\phi_{LCL} \vee \phi_{LCU})$.
- Learnt clauses ϕ_{LCL} and cubes ϕ_{LCU}.
- Q-resolution/consensus to derive learnt clauses/cubes.
- See also our SAT'10 paper.

System Overview (6/7)

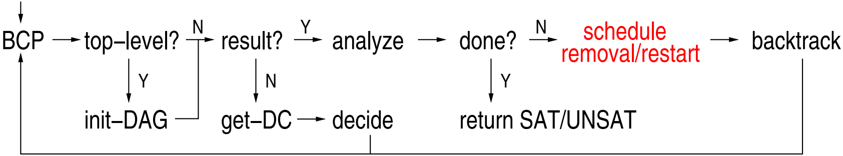


Figure: DepQBF workflow.

Learnt Constraint Removal and Restarts:

- Check each time when adding a new learnt constraint.
- Capacity exhausted: remove half of learnt constraints.
- Heuristically try to keep “useful” constraints, increase capacity.
- Inner-outer restart schedule (like PicoSAT).

System Overview (7/7)

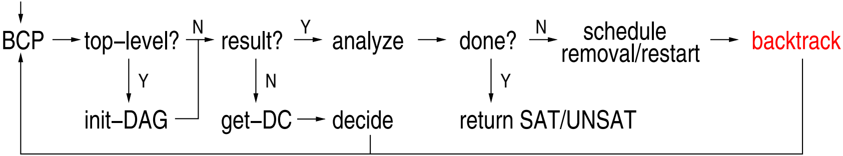


Figure: DepQBF workflow.

Backtracking:

- General (frequent) case: backtrack to asserting level of learnt constraint.
- Special case: backtrack to restart level.

Learnt Constraint Removal

Learnt Constraints: [GNT02, Let02, ZM02, GNT06, BKF95, GS08, ES03, GN02]

- Clauses ϕ_{LCL} and cubes ϕ_{LCU}, stored in doubly-linked lists.
- Initial capacities depend on formula size: $[2500, 10000]$.

Move-To-Front (MTF) Strategy: approximating clause activities.

- Want to keep “used” (i.e. important?) constraints: units, learning.
- Move used constraints C_i to head of list:

$$\{C_1, \dots, C_{i-1}, C_i, C_{i+1}, \dots, C_n\} \overset{MTF(C_i)}{\Longrightarrow} \{C_i, C_1, \dots, C_{i-1}, C_{i+1}, \dots, C_n\}$$

most-recently used least-recently used ⟵ deletion order ↩

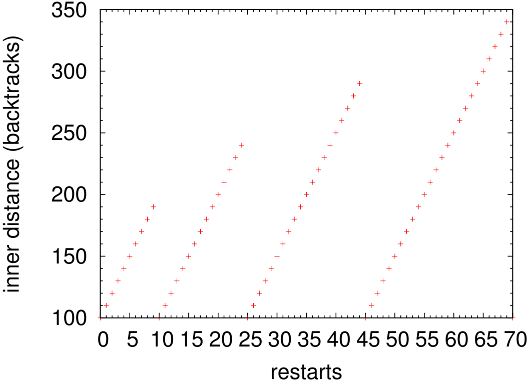
Deletion:

- Capacity exhausted: remove half of constraints, starting at tail of list.
- Least-recently used ones are deleted (hopefully: least-important ones).
- Increase capacity by constant 500.

Restarts (1/2)

Inner-Outer Restart Schedule: when to restart? [Bie08]

- Inspired by PicoSAT: separate inner/outer restarts.
- Inner restart after *i* backtracks, outer restart after *o* inner restarts.
- Initially $i := 100$, $o := 10$.
- Before *i*th ordinary backtrack: jump to *restart level* instead, $i := i + 10$.
- After *o* inner restarts: $i := 100$, $o := o + 5$ (outer restart).



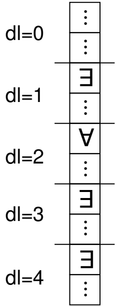
Restarts (2/2)

Restart Level: where to jump to?

- Normally, DepQBF always jumps to asserting level.
- Restart: possibly jump *most-recent universal decision level* instead.
    - Always the longer jump is taken.
- Related to ideas from unrestricted backtracking [BLdSMS05].

Example:

- Assignment stack, in order of decision levels.
- Conflict/solution at level 4.
- Restart is scheduled, where to jump to?



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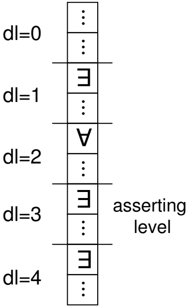
Example:

- Current learnt constraint asserting at level 3.
- Last universal decision at level 2.



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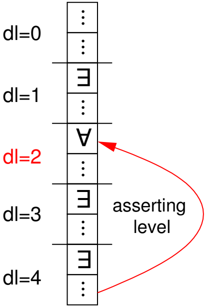
- Restart: take the longer jump.



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- Restart: possibly jump *most-recent universal decision level* instead.
  - Always the longer jump is taken.
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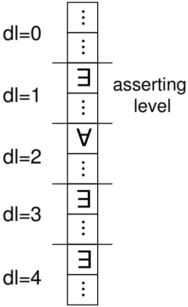
Example:

- Current learnt constraint asserting at level 1.
- Last universal decision at level 2.



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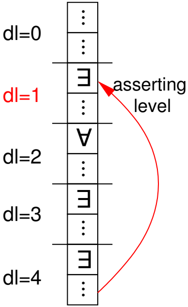

Experiments: QBFEVAL'10 Main Track

	All		*Solved SAT*		*Solved UNSAT*	
	solved	*avg.time*	*solved*	*avg.time*	*solved*	*avg.time*
without preprocessing						
DepQBF	370	337.10	165	54.58	205	20.82
DepQBF-nr	360	352.33	154	51.36	206	24.35
DepQBF-nc	350	384.66	157	107.48	193	28.05
DepQBF-np	345	398.12	141	114.72	204	45.37
DepQBF-ncnr	340	400.24	147	124.10	193	20.19
QuBE7.0-nopp	332	425.44	135	147.71	197	47.27
QuBE6.6-nopp	301	468.51	113	136.48	188	55.27

Table: QBFEVAL'10 main track (568 formulae). Ranking by number of solved formulae.

Setup:

- Ubuntu 9.04, Intel® Q9550@2.83 GHz, 3 GB/900 sec.
- DepQBF: version 0.1 which participated in QBFEVAL'10.

Experiments: QBFEVAL'10 Main Track

	All		*Solved SAT*		*Solved UNSAT*	
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Important:

- Restarts (disabled in DepQBF-nr).

Experiments: QBFEVAL'10 Main Track

	All		*Solved SAT*		*Solved UNSAT*	
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Important:

- Restarts.
- Assignment caching (disabled in DepQBF-nc).

Experiments: QBFEVAL'10 Main Track

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	solved	*avg.time*	*solved*	*avg.time*	*solved*	*avg.time*
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Table: QBFEVAL'10 main track (568 formulae). Ranking by number of solved formulae.

Important:

- Restarts.
- Assignment caching.
- Pure literal detection (disabled in DepQBF-np).

Experiments: QBFEVAL'10 Main Track

	All		*Solved SAT*		*Solved UNSAT*	
	solved	*avg.time*	*solved*	*avg.time*	*solved*	*avg.time*
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Table: QBFEVAL'10 main track (568 formulae). Ranking by number of solved formulae.

Important:

- Restarts.
- Assignment caching.
- Pure literal detection.
- Combining restarts with assignment caching (disabled in DepQBF-ncnr).

Experiments: QBFEVAL’10 Main Track

	All		*Solved SAT*		*Solved UNSAT*	
	solved	*avg.time*	*solved*	*avg.time*	*solved*	*avg.time*
QuBE7.0-pre⇒DepQBF	**424**	**254.23**	**197**	**48.17**	**227**	**23.42**
QuBE7	**414**	**310.29**	**187**	**130.52**	**227**	**58.33**
QuBE6.6	**387**	**341.91**	**168**	**98.97**	**219**	**67.03**
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DepQBF-nr	360	352.33	154	51.36	206	24.35
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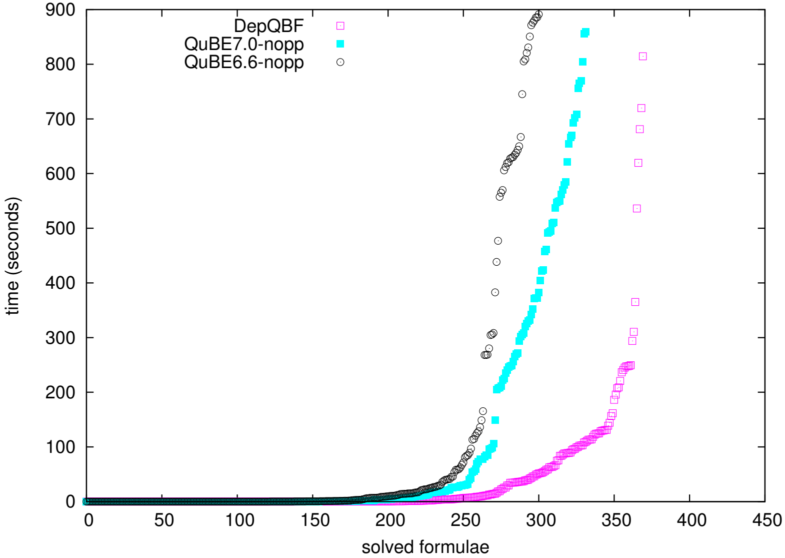
Table: QBFEVAL’10 main track (568 formulae). Ranking by number of solved formulae.

Important:

- Restarts.
- Assignment caching.
- Pure literal detection.
- Combining restarts with assignment caching.
- Preprocessing (**not** part of DepQBF 0.1, disabled in QuBE*-nopp).

Experiments: QBFEVAL'10 Main Track



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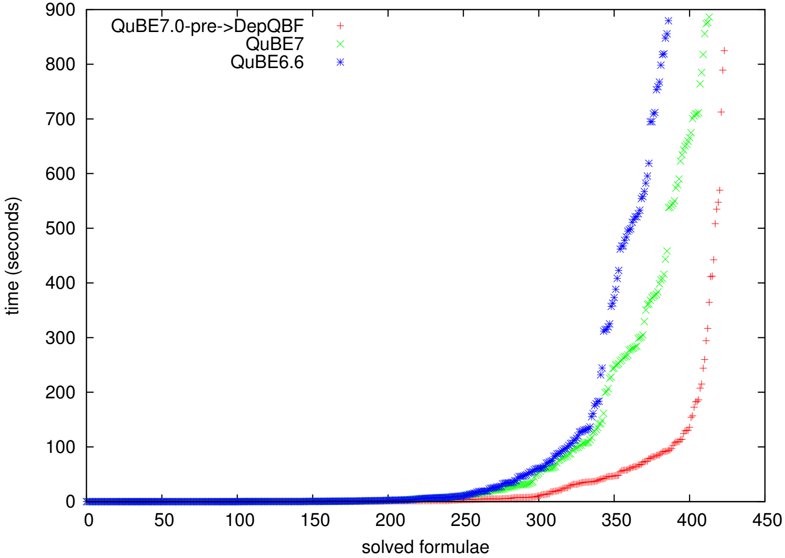

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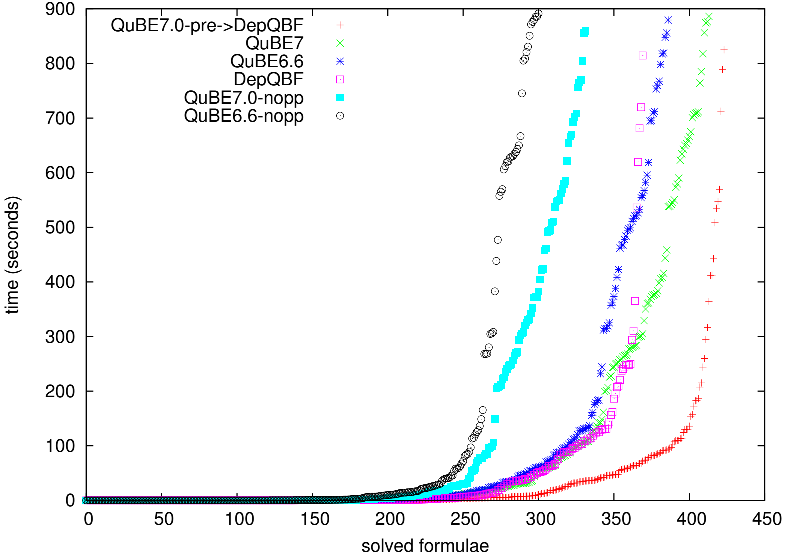

Conclusion

DepQBF: [BLB10]

- Search-based QBF solver with clause- and cube-learning.
- Relaxing prefix order by dependency-DAG for D^{std}.
- Approaches from SAT domain.
- Development:
  - Fuzz testing using QBFuzz: `http://fmv.jku.at/qbfuzz/`
  - Delta-debugging using QBFDD: `http://fmv.jku.at/qbfdd/`
  - Cross-checking against other solvers, mainly QuBE.

Performance:

- Top-ranked solver in QBFEVAL'10.
- DepQBF 0.1 does *not* include preprocessing.
- But: preprocessing is *very* important.

Future Work:

- Preprocessing, parameter tuning, decision heuristics, . . .

DepQBF 0.1 is open source: `http://fmv.jku.at/depqbf/`

[APPENDIX] – BCP: Unit Literals

Unit Clauses: Clause C is unit iff [CGS98, GGN^{+}03, MMZ^{+}01, GNT07]

- no $l \in C$ is true.
- exactly one $l_e \in L_\exists(C)$ is unassigned.
- for all unassigned $l_u \in L_\forall(C)$: $l_u \not\prec l_e$, i.e. $Var(l_u)$, $Var(l_e)$ independent.
- Dependency checking $\prec$ with respect to dependency scheme.
- Dual definition for cubes.

Two-Literal-Watching:

- Watch two unassigned literals $l_1, l_2 \in C$ such that
  (1) either $q(l_1) = q(l_2) = \exists$, or
  (2) $q(l_1) = \forall$, $q(l_2) = \exists$ and $l_1 \prec l_2$.

Watcher Update:

- Dependency checking needed only in case (2).
- Stop when finding satisfying literal.
- No work needed during backtracking.

[APPENDIX] – BCP: Pure Literals (1/2)

Pure Literals (PL): [CGS98, GGN$^+$03, GNT04]

- Variable has only positive/negative literals left.
- Assigning ∀-PLs/∃-PLs can trigger new units/further PLs.
- Drawback: expensive detection in $\phi_{OCL} \wedge (\phi_{LCL} \vee \phi_{LCU})$.

Spurious Pure Literals (SPL):

- Def.: Variable is pure (SPL) if it is pure in original clauses ϕ_{OCL} only.
- SPL-Detection neglects all learnt constraints in $(\phi_{LCL} \vee \phi_{LCU})$.
  - Advantage: more efficient detection.
- Variable might be pure in ϕ_{OCL} but not in $\phi_{OCL} \wedge (\phi_{LCL} \vee \phi_{LCU})$.
  - Drawback: must ignore such SPL-implications in $(\phi_{LCL} \vee \phi_{LCU})$.

Clause Watching:

- Positive/negative occurrences $C(x), C(\overline{x}) \subseteq \phi_{OCL}$.
- Watch two unsatisfied clauses $C_x \in C(x)$ and $C_{\overline{x}} \in C(\overline{x})$.

[APPENDIX] – BCP: Pure Literals (2/2)

Clause Watcher Update:

- Assign $x/\overline{x}$: all clauses in $C(x)/C(\overline{x})$ will be satisfied.
- Update watchers of variables y watching clauses in $C(x)/C(\overline{x})$.

Notification Lists:

- Goal: avoid searching for variables which need watcher update.
- Lists $NL_x/NL_{\overline{x}}$ of variables y watching clauses in $C(x)/C(\overline{x})$.
- Assign $x/\overline{x}$:
  - *exactly* all variables in $NL_x/NL_{\overline{x}}$ must update their watcher.
  - update $NL_x/NL_{\overline{x}}$ of variables x occurring in old and new watched clauses.
- No work needed during backtracking.

[APPENDIX] – Variable Activities

Activity-Based Variable Priority Queue: [MMZ$^+$01, ES03]

- DepQBF: straight-forward generalization of idea from SAT domain.
- Maintain VSIDS score (activity) for each variable.
- Increase activity of variables encountered during learning.
- Periodically down-scale activities.
- Implementation follows MiniSAT 2.
- Decision making: select *candidate* with highest activity.
- Lazy priority queue maintenance (like in MiniSAT):
  - Discard assigned variables and non-candidates on the fly upon removal.

[APPENDIX] – Assignment Caching

Also called: Phase Saving [PD07]

- DepQBF: straight-forward generalization of idea from SAT-domain.
- Each variable has a cached assignment (possibly undefined).
- All assignments (unit, pure literals, decisions) update cache.
- Decision variables: assign cached value, if any.
- No distinction between different quantifiers.

[APPENDIX] – Experiments: Suite *mqm*

Suite mqm (136 formulae)		
	solved	*avg.time*
DepQBF	136	39.83
QuBE7	117	306.43
QuBE7.0-nopp	115	304.82
QuBE6.6	100	393.93
QuBE6.6-nopp	97	399.55

Table: Solvers sorted by number of solved formulae.

Benchmark Suite *mqm*:

- Minimal Query Inseparability Module Extraction in DL-Lite.
- Newly submitted to QBFEVAL'10 by Roman Kontchakov.
- As the only solver, DepQBF solved entire suite in QBFEVAL'10.

[APPENDIX] – Experiments: Pairwise Comparison

QBFEVAL'10: solved formulae only						
	∩		SAT-∩		UNSAT-∩	
solved	328		132		196	
avg.time	84.97	**21.87**	140.16	**32.43**	47.81	**14.75**
QBFEVAL'10: unique results						
	⇔		SAT-⇔		UNSAT-⇔	
solved	**86**	42	**55**	33	**31**	9

Table: QuBE7 (left columns) vs. DepQBF (right columns).

QBFEVAL'10: solved formulae only						
	∩		SAT-∩		UNSAT-∩	
solved	308		115		193	
avg.time	80.14	**17.49**	114.17	**23.23**	59.86	**14.07**
QBFEVAL'10: unique results						
	⇔		SAT-⇔		UNSAT-⇔	
solved	**79**	62	**53**	50	**26**	12

Table: QuBE6.6 (left columns) vs. DepQBF (right columns).

- M. Benedetti.
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